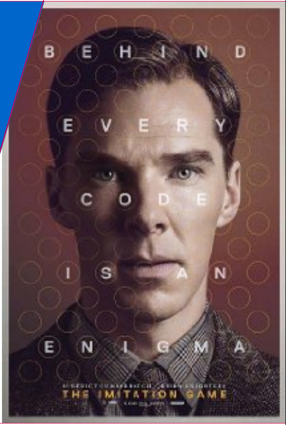
Executable Behaviours and the π-Calculus

Bas Luttik   Fei Yang



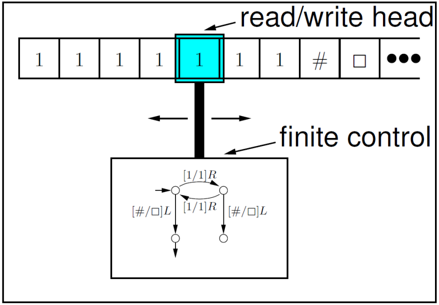
Technische Universiteit
Eindhoven
University of Technology

June 5, 2015

Where innovation starts

Outline

Turing machine and Computers



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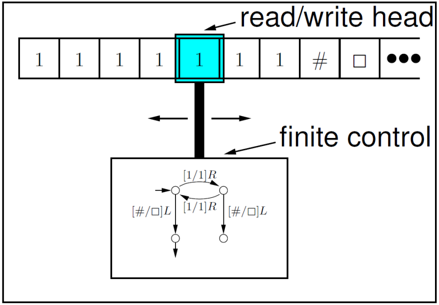

Church Turing Thesis: every computable function can be computed with a Turing Machine

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The CT thesis is sometimes paraphrased as:
"A TM can do everything a real computer can do"

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Concurrency Theory is introduced to study such systems.

- Interaction: between parallel components

Computation in Concurrency Theory

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- Non-termination : infinitely long execution sequence (divergence)

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Concurrency +Computability

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Concurrency Theory + CT Thesis?

Concurrency +Computability = Executability

Absolute Expressiveness

Given a process calculus:

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Outline

From Computability to Executability

Evaluating Expressiveness w.r.t. Executability

Expressiveness of the π-Calculus

- Reactive Turing Powerfulness of the π-Calculus
- Executability of the π-Calculus Processes

Outline

Reactive Turing Machines

$\mathcal{A}$: set of actions; τ: a special action ($\notin \mathcal{A}$), for unobservable actions.

$\mathcal{A}_\tau = \mathcal{A} \cup \{\tau\}$.

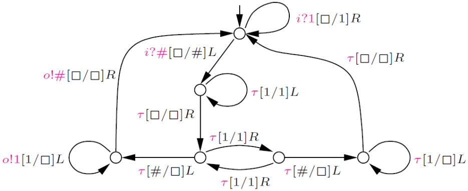
A reactive Turing machine (RTM) is a classical Turing machine with an action from some set $\mathcal{A}_\tau$ associated with every transition.

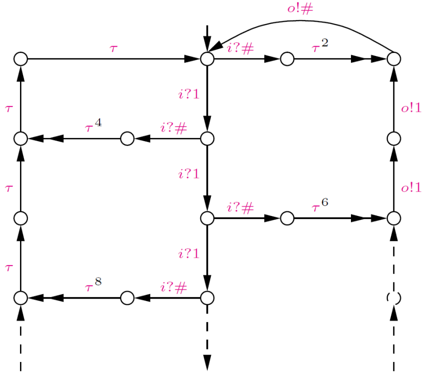
So RTMs have two types of transitions:

1. $s \xrightarrow{a[d/e]M} t$ means "externally observable, as execution of a"
2. $s \xrightarrow{\tau[d/e]M} t$ means "internal, unobservable transition"

M is ether "moving left" or "moving right"

Labelled Transition System of an RTM





We associate with every configuration (control state, tape instance) a state, and associate with every execution step a labelled transition.

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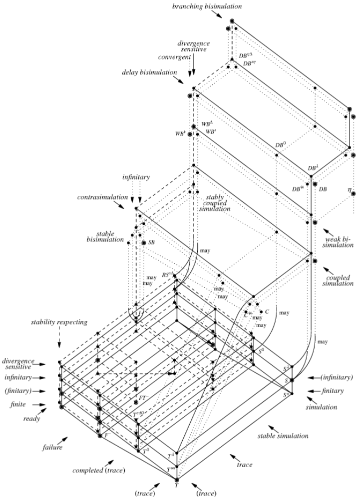

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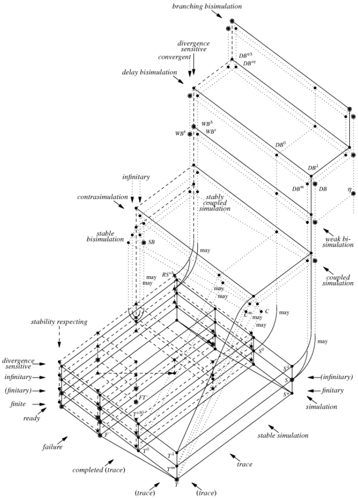
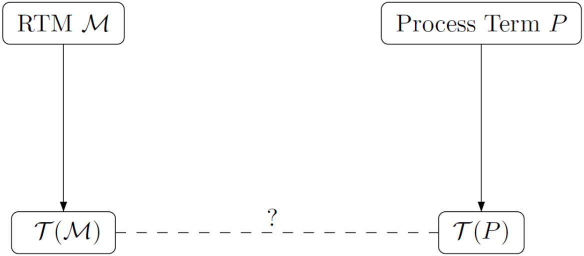
We start from (divergence-preserving) branching bisimilarity



Evaluating Expressiveness

$$\begin{tikzcd} \text{RTM } \mathcal{M} \arrow[d] & \text{Process Term } P \arrow[d] \\ \mathcal{T}(\mathcal{M}) \arrow[r, dash, dashed, "?"] & \mathcal{T}(P) \end{tikzcd}$$

Evaluating Expressiveness



1. Can we specify every executable LTS by the LTS associated with *P*? (reactive Turing powerfulness?)

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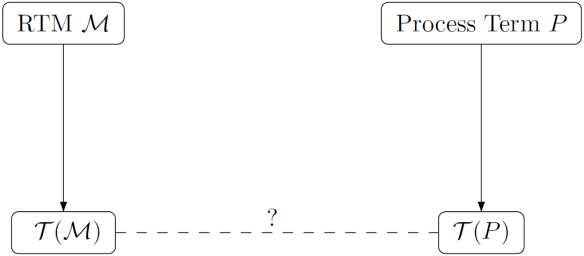

1. Can we specify every executable LTS by the LTS associated with *P*? (reactive Turing powerfulness?)

2. Is every LTS associated with the process specifiable by *P* executable? (executability)

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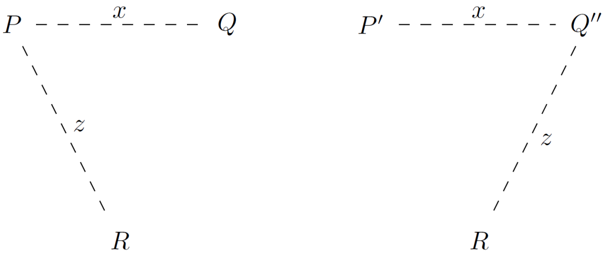
We presuppose a countably infinite set $\mathcal{N}$ of names.

The prefixes, processes and summations of the π-calculus are, respectively, defined by the following grammar:

$$\begin{aligned}
\pi &:= \overline{x}y \mid x(z) \mid \tau \qquad (x, y, z \in \mathcal{N}) \\
P &:= M \mid P \mid P \mid (z)P \mid !P \\
M &:= 0 \mid \pi.P \mid M + M \quad .
\end{aligned}$$

Link Mobility

Suppose $P = \overline{x}z.P'$, $Q = x(y).Q'$.
Then $(z)(P \mid R) \mid Q \xrightarrow{\tau} P' \mid (z)(R \mid Q'')$, where $Q'' = \{z/y\}Q'$.



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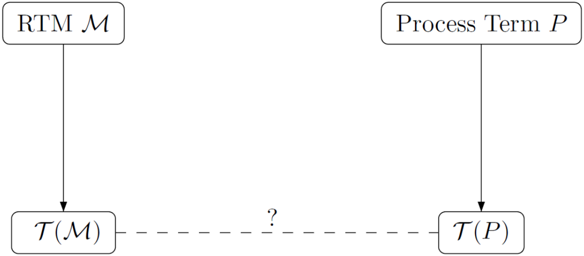

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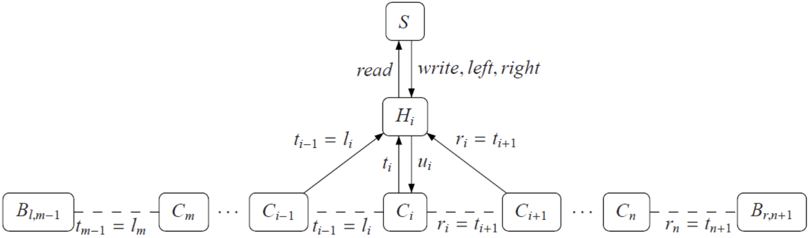
Specification of an RTM

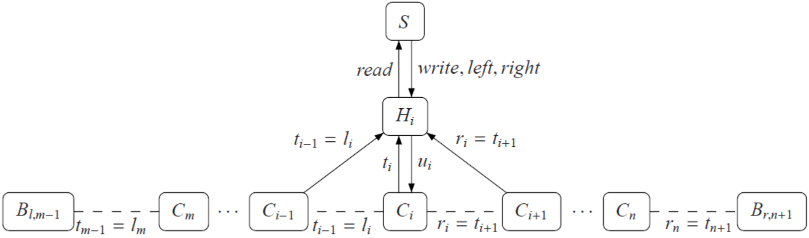
The specification contains two parts:

1. A generic process to specify the tape of a machine, and
2. a bunch specific processes for transition rules.



Tape

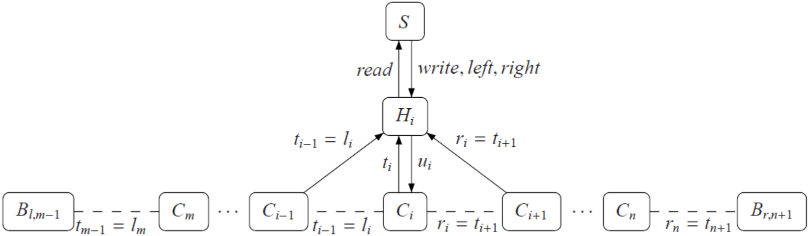
1. Tape head: read, write, move
2. Cells: an ordered sequence to record data
3. Generator: a facility to generate new cells



Control of the machine

The transition rules of RTMs are of the form:

$$s \xrightarrow{a[d/e]M} t$$



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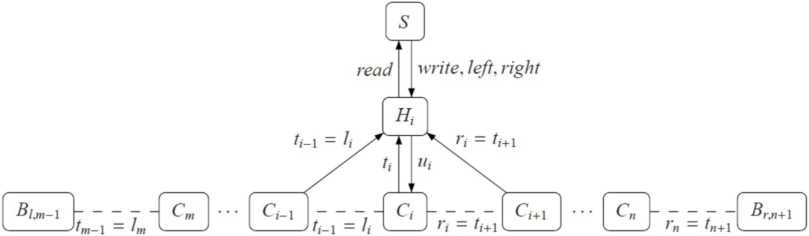
The state s and data d determine the set of subsequent transitions.



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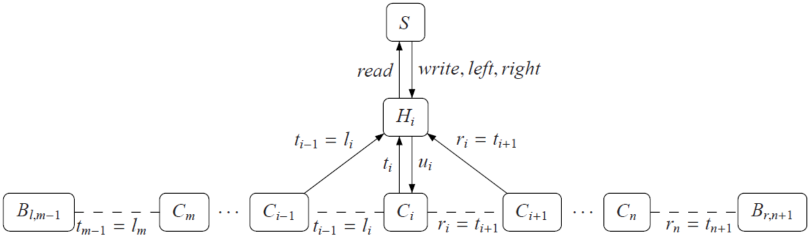
$$S_{s,d} \stackrel{\text{def}}{=} \sum_{(s,d,a,e,m,t)\in \longrightarrow_{\mathcal{M}}} a.\overline{write}\, e.\overline{m}.read(f).S_{t,f}$$



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π-calculus is reactive Turing powerful modulo divergence-preserving branching bisimilarity.

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Simulating π-processes with RTMs

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Two choices:

Extend the formalism of RTMs to an infinite set of actions.

Restrict the π-calculus with finitely many names.

Extend RTMs with infinitely many actions

An infinite alphabet of data symbols or control states is required.

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Theorem

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Not realistic!

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An alternative semantics

For a finite set of names $\mathcal{N}'$ and a π-term P, we define the labelled transition system of P over $\mathcal{N}'$ as $\mathcal{T}(P) \upharpoonright \mathcal{N}'$, where

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$\mathcal{T}(P) \restriction \mathcal{N}'$ actually collects exactly all the behaviour of P regarding to $\mathcal{N}'$.

Executability of the π-calculus

Theorem

Every closed π-term with finitely many observable names is executable up to branching bisimilarity, but there exist closed π-terms with finitely many observable names that are not executable up to divergence-preserving branching bisimilarity.

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It is executable modulo branching bisimilarity, and but not modulo divergence-preserving branching bisimilarity.

Conclusion

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- A framework to evaluate the expressiveness for a model of concurrency
- An application to the π-calculus
  - Reactive Turing powerfulness
  - Executability

Thank You!