Reversible Barbed Congruence on Configuration Structures

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[3]ANR-14-CE25-0005 ELICA & ANR-11-INSE-0007 REVER

5 June 2015

$$P = (a.b.0) + (b.a.0)$$

$$Q = (a.0)|(b.0)$$

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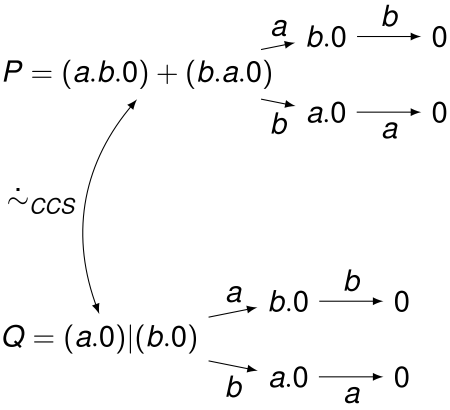
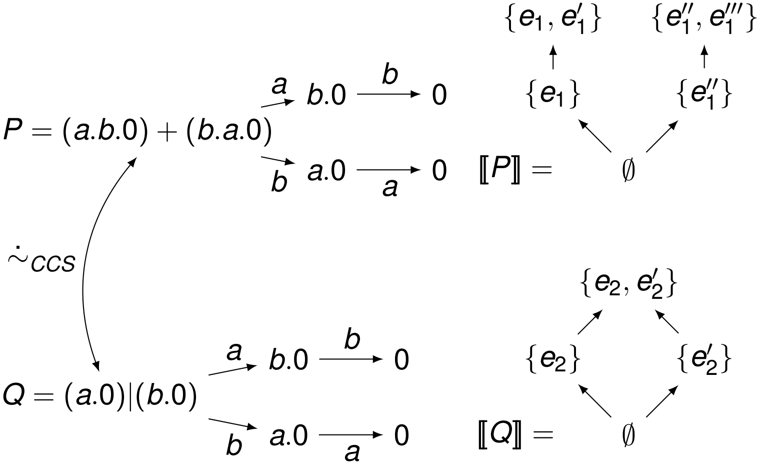
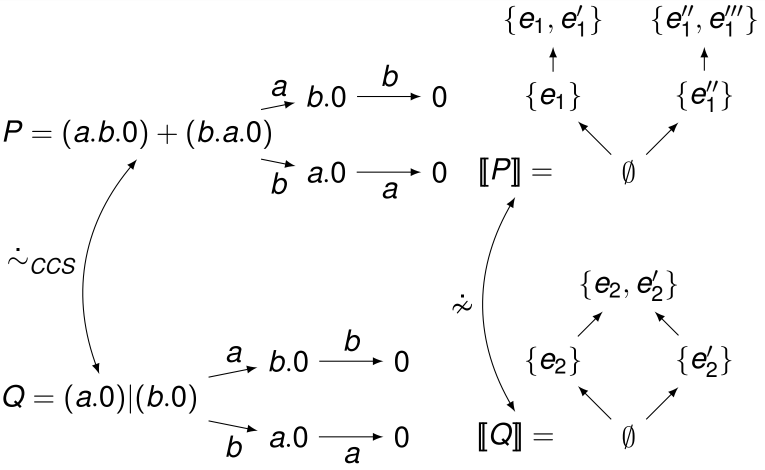


CCS



CCS

configuration structures



CCS

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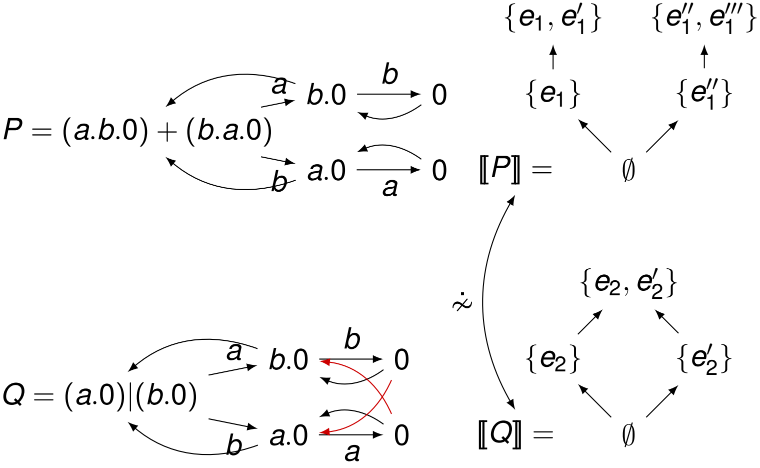

RCCS

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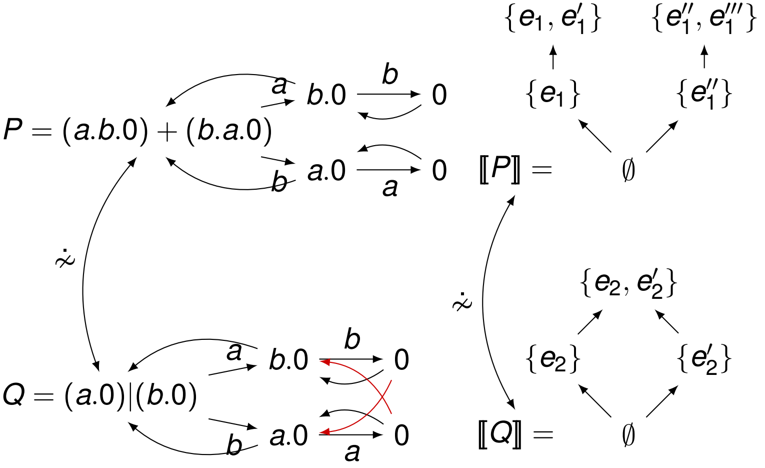

RCCS

configuration structures

labelled equivalence $\Leftrightarrow$ contextual equivalence

$P \sim^{\tau} Q \Leftrightarrow$ if for every *context* C, $C[P]$ and $C[Q]$ have the same *observables*.

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Context $C := [\,] \parallel \alpha.C \parallel C + P \parallel C|P$.

Observable

- the *reductions*
- the *barbs*: $P \downarrow_{\alpha}$ if there exists P' such that $P \xrightarrow{\alpha} P'$.

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Strong barbed congruence

$$P \dot{\sim}^{\tau}{}_{CCS}\ Q \Leftrightarrow \begin{cases} P \xrightarrow{\tau} P' \Rightarrow Q \xrightarrow{\tau} Q' \wedge P' \dot{\sim}^{\tau}{}_{CCS}\ Q' \\ P \downarrow_{\alpha} \Rightarrow Q \downarrow_{\alpha} \\ Q \dot{\sim}^{\tau}{}_{CCS}\ P \end{cases}$$

$$P \sim^{\tau}{}_{CCS}\ Q \Leftrightarrow \forall C, C[P] \dot{\sim}^{\tau}{}_{CCS}\ C[Q]$$

labelled equivalence $\Leftrightarrow$ contextual equivalence

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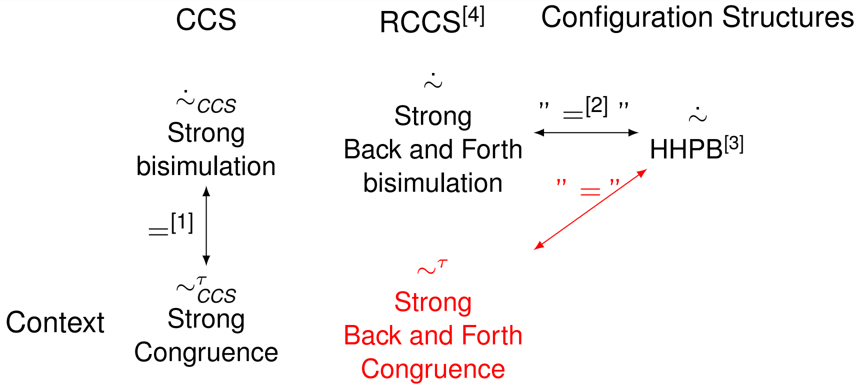
contextual characterisation of hereditary history-preserving bisimulation (hhpb)?



[1] Milner and Sangiorgi (1992). "Barbed Bisimulation"
[2] Phillips and Ulidowski (2012). "Reversibility and Models for Concurrency"
[3] Bednarczyk (1991). "Hereditary history preserving bisimulation"
[4] Danos and Krivine (2004). "Reversible Communicating Systems"

$$R, S := m \triangleright P \ \parallel \ R|R \qquad \text{(RCCS processes)}$$
$$m := \emptyset \ \parallel \ \curlyvee .m \ \parallel \ \langle i, a, P \rangle.m \ \parallel \ \langle i, a \rangle.m \qquad \text{(Memories)}$$

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Example

$$\emptyset \triangleright (a.P + b.Q)|(c.\overline{a}.P')$$

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&\xrightarrow{3:\tau}(\langle 3, a, b.Q\rangle.\curlyvee.\emptyset \triangleright P)|(\langle 3, \overline{a}\rangle.\langle 2, c\rangle.\curlyvee.\emptyset \triangleright P')
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Contextual equivalence

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Strong back-and-forth barbed bisimulation

$$R \dot{\sim}^{\tau} S \Leftrightarrow \begin{cases} R \twoheadrightarrow^{\tau} R' \Rightarrow S \twoheadrightarrow^{\tau} S' \wedge R' \dot{\sim}^{\tau} S' \\ R \rightsquigarrow^{\tau} R' \Rightarrow S \rightsquigarrow^{\tau} S' \wedge R' \dot{\sim}^{\tau} S' \\ R \downarrow_{\alpha} \Rightarrow S \downarrow_{\alpha} \\ S \dot{\sim}^{\tau} R \end{cases}$$

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Strong back-and-forth barbed congruence

$\sim^{\tau} = \dot{\sim}^{\tau}$ closed by *context*

Origin of a process

$$O_R = P \text{ such that } \emptyset \rhd P \rightarrow^* R$$

Simplest case: contexts and processes with an empty memory.

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Example

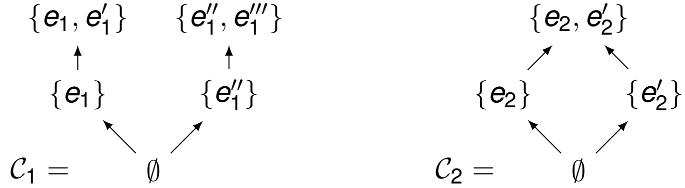
$$R = \langle i, b, a.P\rangle.\emptyset \triangleright Q \qquad S = \langle j, a, b.Q\rangle.\emptyset \triangleright P$$

$O_R = O_S = \emptyset \triangleright a.P + b.Q$, so $O_R \sim^{\tau} O_S$ but $R \dot{\nsim}^{\tau} S$!

Labelled configuration structure[5]

$\mathcal{C} = \langle E, C, \ell \rangle$ with $C \subset \mathcal{P}(E)$ and $\ell : E \rightarrow$ labels.

Example



$\ell_1(e_1) = \ell_1(e_1''') = a$
$\ell_1(e_1') = \ell_1(e_1'') = b$

$\ell_2(e_2) = a$
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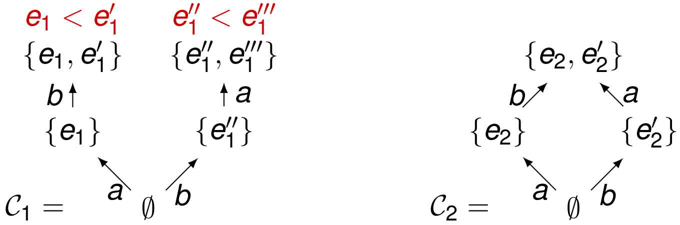

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$(\emptyset, \emptyset, \emptyset) \in \mathcal{R}$, and for $x_i \in C_i, e_i \in E_i, (x_1, x_2, f) \in \mathcal{R} \Rightarrow$

f label and order preserving bijection

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$x_1 \rightarrow^{\alpha} x_1 \cup \{e_1\} \implies$
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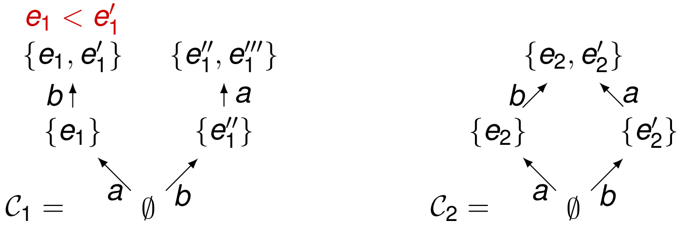

$$\emptyset, \emptyset, \emptyset$$

$$\{e_1\}, \{e_2\}, e_1 \leftrightarrow e_2$$

$$\{e_1, e'_1\}, \{e_2, e'_2\}, e'_1 \leftrightarrow e'_2$$

Reversible Configuration Structures

$$O_R = P \text{ such that } \emptyset \triangleright P \underbrace{\xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_i}}_{x_R = \{e_1, \ldots, e_i\}} R \qquad [\![R]\!] = ([\![O_R]\!], x_R)$$

We can consider only forward transitions.

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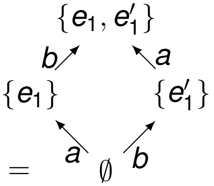
Example

$[\![P]\!] =$



$$P = a.0 | b.0$$

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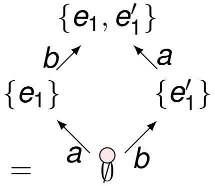

$$\emptyset \triangleright P \equiv \curlyvee.\emptyset \triangleright a.0 \mid \curlyvee.\emptyset \triangleright b.0$$

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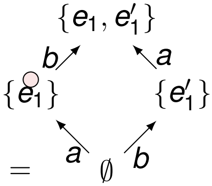

$$\emptyset \rhd P \equiv \curlyvee.\emptyset \rhd a.0 | \curlyvee.\emptyset \rhd b.0$$

$$\xrightarrow{1:a} \langle 1, a \rangle. \curlyvee.\emptyset \rhd 0 | \curlyvee.\emptyset \rhd b.0$$

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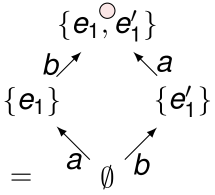

$$\emptyset \rhd P \equiv \curlyvee.\emptyset \rhd a.0 | \curlyvee.\emptyset \rhd b.0$$
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Operational correspondence

let $[\![R]\!] = (\mathcal{C}, x)$:

$$R \xrightarrow{i:\alpha} S \Rightarrow (\mathcal{C}, x) \xrightarrow{\alpha} (\mathcal{C}, x \cup \{e\})$$

$$(\mathcal{C}, x) \xrightarrow{\alpha} (\mathcal{C}, x \cup \{e\}) \Rightarrow R \xrightarrow{i:\alpha} S$$

where $[\![S]\!] = (\mathcal{C}, x \cup \{e\})$, $\ell(e) = \alpha$ and the similarly for $\rightsquigarrow$.

RCCS		Configuration Structures
$R \sim^{\tau} S$		$([\![O_R]\!], x_R)\text{''} \sim \text{''}([\![O_S]\!], x_S)$
$O_R \sim^{\tau} O_S$	$\longleftrightarrow$ $\text{''} = \text{''}$	$[\![O_R]\!] \dot{\sim} [\![O_S]\!]$

RCCS | Configuration Structures

$R \sim^{\tau} S$ $\qquad$ $([\![O_R]\!], x_R)\text{''} \sim \text{''}([\![O_S]\!], x_S)$

$O_R \sim^{\tau} O_S \;\overset{\text{''}=\text{''}}{\longleftrightarrow}\; [\![O_R]\!] \mathrel{\dot\sim} [\![O_S]\!]$

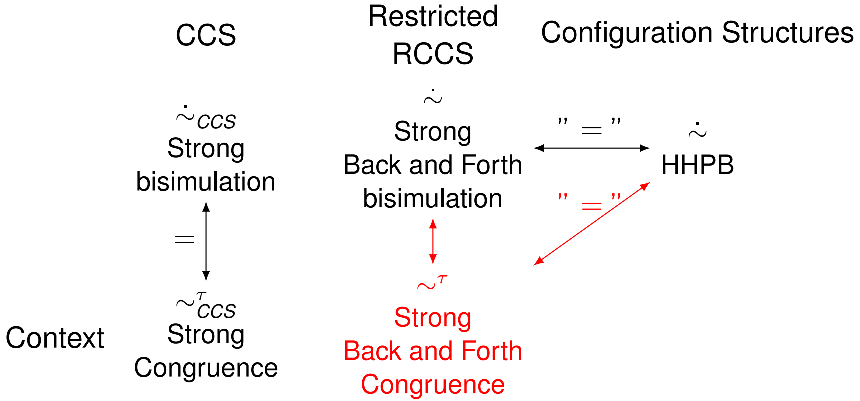
$O_R \sim^{\tau} O_S \;\Longleftrightarrow\; [\![O_R]\!] \sim^{\tau} [\![O_S]\!]$

$[\![O_R]\!] \mathrel{\dot\sim} [\![O_S]\!] \;\Updownarrow\; [\![O_R]\!] \sim^{\tau} [\![O_S]\!]$

+ inductive approximations of hhpb

contextual characterisation of hhpb

$[\![O_R]\!] \mathrel{\dot\sim} [\![O_S]\!] \Leftrightarrow$ for every context C, $[\![C[O_R]]\!] \mathrel{\dot\sim} [\![C[O_S]]\!]$.


CCS
Restricted
RCCS
Configuration Structures
$\dot{\sim}_{CCS}$
Strong
bisimulation
$\dot{\sim}$
Strong
Back and Forth
bisimulation
" = "
$\dot{\sim}$
HHPB
" = "
=
Context
$\sim^{\tau}_{CCS}$
Strong
Congruence
$\sim^{\tau}$
Strong
Back and Forth
Congruence

CCS

Restricted RCCS

Configuration Structures

$\dot{\sim}_{CCS}$
Strong bisimulation

(↕ $=$)

Context: $\sim^{\tau}_{CCS}$
Strong Congruence

$\dot{\sim}$
Strong Back and Forth bisimulation

(↕)

$\sim^{\tau}$
Strong Back and Forth Congruence

(Strong Back and Forth bisimulation ↔ " $=$ " HHPB)

(Strong Back and Forth Congruence ↔ " $=$ " HHPB)

$\dot{\sim}$
HHPB

Future work

More general context for RCCS

Weak case

What to observe? directions?