The LTS WorkBench

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Motivation

You are working on **LTS-based** models for concurrency and **observational relations**

You want to **validate** some theory, on LTS generated by **different calculi**, or maybe just **explore the transitions** of a process

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- **manipulate** them (**compose** them, let them **synchronise**, **filter out** some parts,. . .)
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- you try to **encode your theory** in the process/logic language supported by some tool (e.g. mCRL2, CADP)
- **otherwise**, you implement it **directly**

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What if you are dealing with **(possibly) infinite-state** LTSs and processes, arising e.g. from **recursion, parallelism, unbounded buffers**?

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We were in this situation, but wanted to **avoid yet another ad-hoc implementation**

- we ended up with `LTSwb`

A reusable semantic framework

Observation: different process calculi have **similar operators**

- sequential execution
- choice
- parallel composition
- synchronisation
- . . .

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Idea:

1. define **as many operators as possible** at a **semantic, syntax-independent level**
2. mix & match to **cook your process calculus** — **if** needed!

Example: a calculus with sequencing

We want to implement and study a process calculus C
with the usual **sequential composition** `(p seq q)`

$$\frac{\mathtt{p} \xrightarrow{\ell} \mathtt{p}'}{(\mathtt{p}\ \ \mathtt{seq}\ \ \mathtt{q}) \xrightarrow{\ell} (\mathtt{p}'\ \ \mathtt{seq}\ \ \mathtt{q})} \qquad \frac{\mathtt{p} \not\to \quad \mathtt{q} \xrightarrow{\ell} \mathtt{q}'}{(\mathtt{p}\ \ \mathtt{seq}\ \ \mathtt{q}) \xrightarrow{\ell} (\mathtt{p}\ \ \mathtt{seq}\ \ \mathtt{q}')}$$

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- what if $\mathtt{p}$,$\mathtt{q}$ come e.g. from **execution logs**?

Can we implement such a composition upon a **reusable syntax-independent** foundation?

Definitions

An **LTS** is a triple $\big(\Sigma, \Lambda, \mathcal{R}\big)$ where:

- $\Sigma = \{p, q, r, \ldots\}$ is the set of **states**
- $\Lambda = \{\ell_1, \ell_2, \ldots\}$ is the set of **labels**
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$$(\mathbb{L}, \mathrm{p})(\ell) \coloneqq \Big\{(\mathbb{L}, \mathrm{p}') \;\Big|\; (\mathbb{L}, \mathrm{p}) \xrightarrow{\ell} (\mathbb{L}, \mathrm{p}')\Big\}$$

LTS operators: a (boring) example

The **union of LTSs** $\mathbb{L}_1 = \left(\Sigma_1, \Lambda_1, \mathcal{R}_1\right)$ and $\mathbb{L}_2 = \left(\Sigma_2, \Lambda_2, \mathcal{R}_2\right)$ is:

$$\mathbb{L}_1 \cup \mathbb{L}_2 \ \coloneqq \ \Big(\Sigma_1 \cup \Sigma_2,\ \ \Lambda_1 \cup \Lambda_2,\ \ \mathcal{R}_1 \cup \mathcal{R}_2 \Big)$$

Sequencing of relations

Let $\mathcal{R}_1 \subseteq \big(\Sigma_1 \times (\Lambda_1 \times \Sigma'_1)\big)$ and $\mathcal{R}_2 \subseteq \big(\Sigma_2 \times (\Lambda_2 \times \Sigma'_2)\big)$

The **sequencing of** $\mathcal{R}_1$ **and** $\mathcal{R}_2$ is the relation

$$\mathcal{R}_1 \,;\mathcal{R}_2 \subseteq \Big((\Sigma_1 \times \Sigma_2) \times \big((\Lambda_1 \cup \Lambda_2) \times (\Sigma'_1 \times \Sigma'_2)\big)\Big)$$

inductively defined by the rules:

$$\frac{\big(p, (\ell, p')\big) \in \mathcal{R}_1}{\Big((p, q),\ \big(\ell, (p', q)\big)\Big) \in \mathcal{R}_1 \,;\mathcal{R}_2} \qquad \frac{\mathcal{R}_1(p) = \emptyset \quad \big((q, (\ell, q')\big) \in \mathcal{R}_2}{\Big((p, q),\ \big(\ell, (p, q')\big)\Big) \in \mathcal{R}_1 \,;\mathcal{R}_2}$$

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Equivalently:

$$\Big(\mathcal{R}_1 \,;\mathcal{R}_2\Big)\big((p, q)\big) \;=\; \begin{cases} \big\{\big(\ell, (p', q)\big) \mid \big(\ell, p'\big) \in \mathcal{R}_1(p)\big\} & \text{if } \mathcal{R}_1(p) \neq \emptyset \\ \big\{\big(\ell, (p, q')\big) \mid \big(\ell, q'\big) \in \mathcal{R}_2(q)\big\} & \text{otherwise} \end{cases}$$

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The **sequencing of processes** $(\mathbb{L}_1, p)$ **and** $(\mathbb{L}_2, q)$ is:

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I.e., $(\mathbb{L}_1, \mathrm{p}) \;; (\mathbb{L}_2, \mathrm{q})$ **observationally behaves** as p in $\mathbb{L}_1$, *and then* as q in $\mathbb{L}_2$

From semantic to syntactic sequencing

Back to our calculus C, with sequential composition `(p seq q)`

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Desideratum: $\left(\mathbb{L}_{\mathrm{C}}, \texttt{(p seq q)}\right) \simeq \left(\mathbb{L}_{\mathrm{C}} \,;\mathbb{L}_{\mathrm{C}}, (\mathtt{p}, \mathtt{q})\right)$

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We can define the LTS $\mathbb{L}_{\mathrm{C}}$ so that:

$$\Big(\mathbb{L}_{\mathrm{C}}, \texttt{(p seq q)}\Big)(\ell) = \Big\{\Big(\mathbb{L}_{\mathrm{C}}, \texttt{(p}' \texttt{ seq q}'\texttt{)}\Big) \;\Big|\; (\mathrm{p}', \mathrm{q}') \in \Big(\mathbb{L}_{\mathrm{C}} \,;\, \mathbb{L}_{\mathrm{C}}, (\mathrm{p}, \mathrm{q})\Big)(\ell)\Big\}$$

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Which means:

$$\mathcal{R}_{C}(\texttt{(p seq q)}) = \left\{ (\ell, \texttt{(p}' \texttt{ seq q}'\texttt{)}) \;\middle|\; (\ell, (\mathrm{p}', \mathrm{q}')) \in \left(\mathcal{R}_{C} \,;\, \mathcal{R}_{C}\right)((\mathrm{p}, \mathrm{q})) \right\}$$

Summing up

Syntactic sequencing

$\Uparrow$

Process sequencing

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LTS sequencing

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Relational sequencing

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Syntactic **operator**

⇑

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where “operator” may be
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. . . and this is **how our tool works**

Introducing `LTSwb`

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- advanced **type system**
- access to **JVM libraries**
- **eager** language with `lazy val`ues

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`LTSwb` **features**:

- **purely semantic**: no privileged language for processes
- **generic**: parametric on state/label types and synchronisation
- **lazy**: only generates states and transitions when needed

Internals

- `Set[A]` with `.contains(x)`
- `Relation[A,B]` with `.apply(x:A): Set[B]`
- `Relation3[A,B,C]` with `.apply(x:A): Relation[B,C]`
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Defining LTSs (and processes)

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                  (2, ("+", 3)), (2, ("-", 1))))
val l2 = LTS(List(("p1", ("!a", "p2")),
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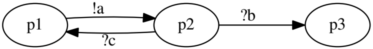

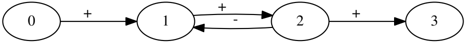

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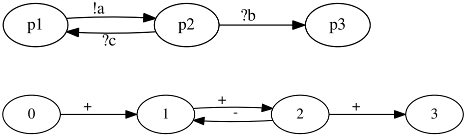
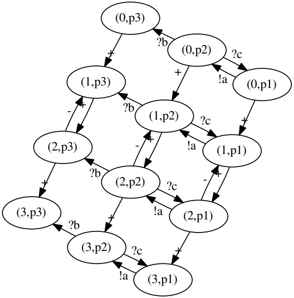


`(l1 ||| l2).toDot` is:



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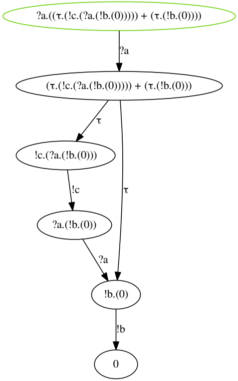
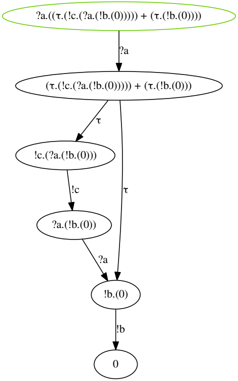
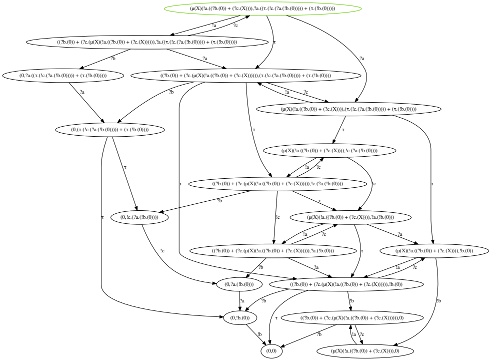


CCS processes

```
// Parses the CCSTerm from String
val ccs1 = CCS.process("rec(X)(!a.(?b + ?c.X))")
// Shorthand. "t" is the internal action
val ccs2 = CCS("?a.(t.!c.?a.!b + t.!b)")
```





The CCS semantics

```
object CCSSemantics
extends FiniteBranchingRelation3[CCSTerm, CCSPfx, CCSTerm] {
  override def apply(s: CCSTerm) = s match {
    case CCSNil() => EmptyRelation()
    case CCSSeq(prefix, cont) => Relation(List((prefix, cont)))
    case CCSPlus(term1, term2) => this(term1) | this(term2)
    case CCSPar(term1, term2) => {
      (CCS ||| CCS).relation((term1, term2)).iso(
        (t:Tuple2[CCSTerm,CCSTerm]) => CCSPar(t._1, t._2),
        (t:CCSPar) => (t.term1, t.term2) )
    }
    case CCSRec(_,_) => this(s.unfold)
    case CCSVar(_) => EmptyRelation() // Free rec variable
    case CCSDel(n, b) => {
      CCS.del(CCSInPfx(n)).del(CCSOutPfx(n)).relation(b).iso(
        (t:CCSTerm) => CCSDel(n, t),  (t:CCSDel) => t.body )
    }
  }
}
```

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. . . and also with **(async) CCS**, e.g.:

$$?a \mid \mathrm{rec}_X\Big(?b.\big(\tau.!c + ?d.X\big)\Big)[!e.!d]$$

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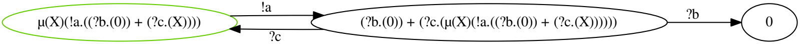
Can we **generalise** such an **"async transformation"**?

Semantic asynchrony

`ccs1.toDot()`



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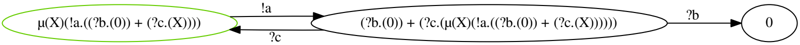
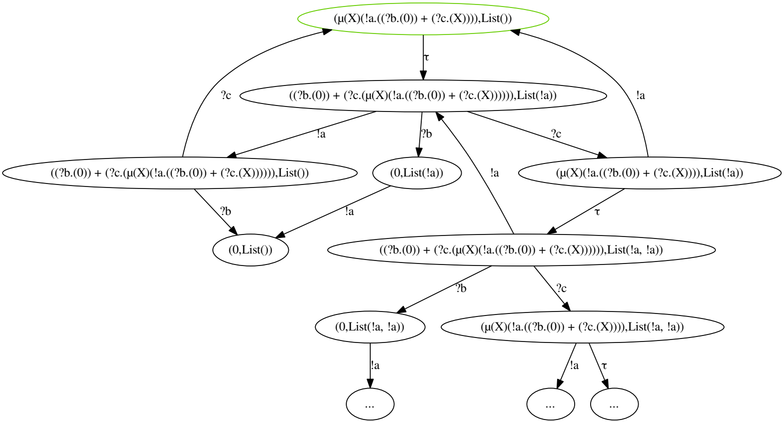


`ccs1.async.toDot(maxDepth=Finite(4))`



Relations

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val p1 = CCS("!a.!b.rec(X)(?c.?c.X)")
val p2 = CCS("!a.!b") seq CCS("rec(Y)(?c.Y)") seq CCS("!d")
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Are `p1` and `p2` **observationally equivalent**?

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(Fernandez and Mounier. *Verifying Bisimulations “On the Fly”*, FORTE 1990)

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Similar machinery for **simulation**, client/server **progress**, client/server **(I/O) compliance**, . . .

Conclusions

`http://tcs.unica.it/software/ltswb`

- initial phases of development
- **praxis-theory-praxis** loop:
    - sticking to theory **reduces code** and **improves reusability**
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Ongoing and future work

- **formalise** the relational → LTS → process → syntax way
- **larger library** of process languages and relations
- **multiparty** interactions via **decorations**? (see PCCS)
- **value-passing** and **time**
- interface with **Gephi**

Thanks!

(Questions?)