Lattices for Space/Belief and Extrusion/Utterance

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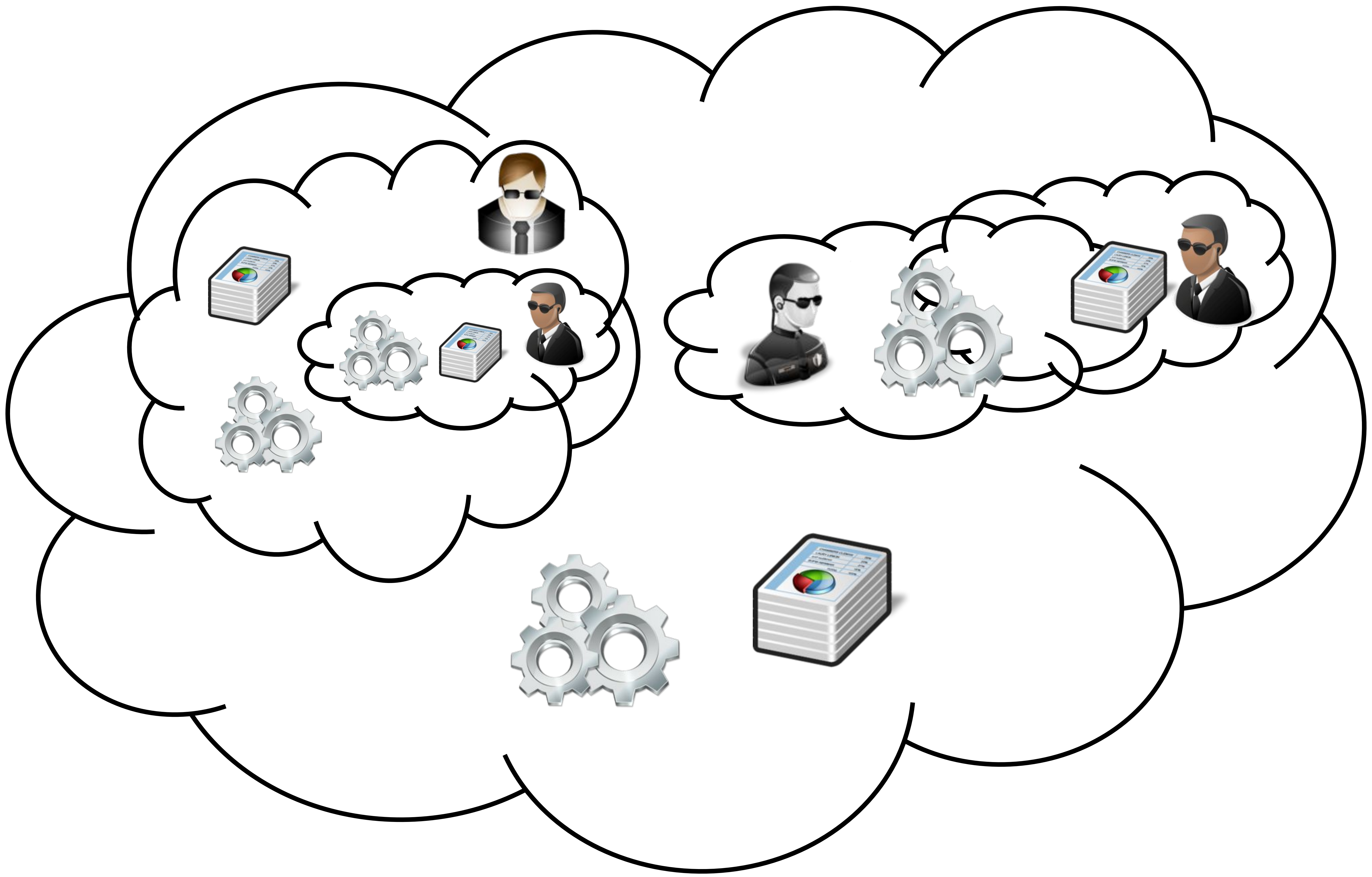
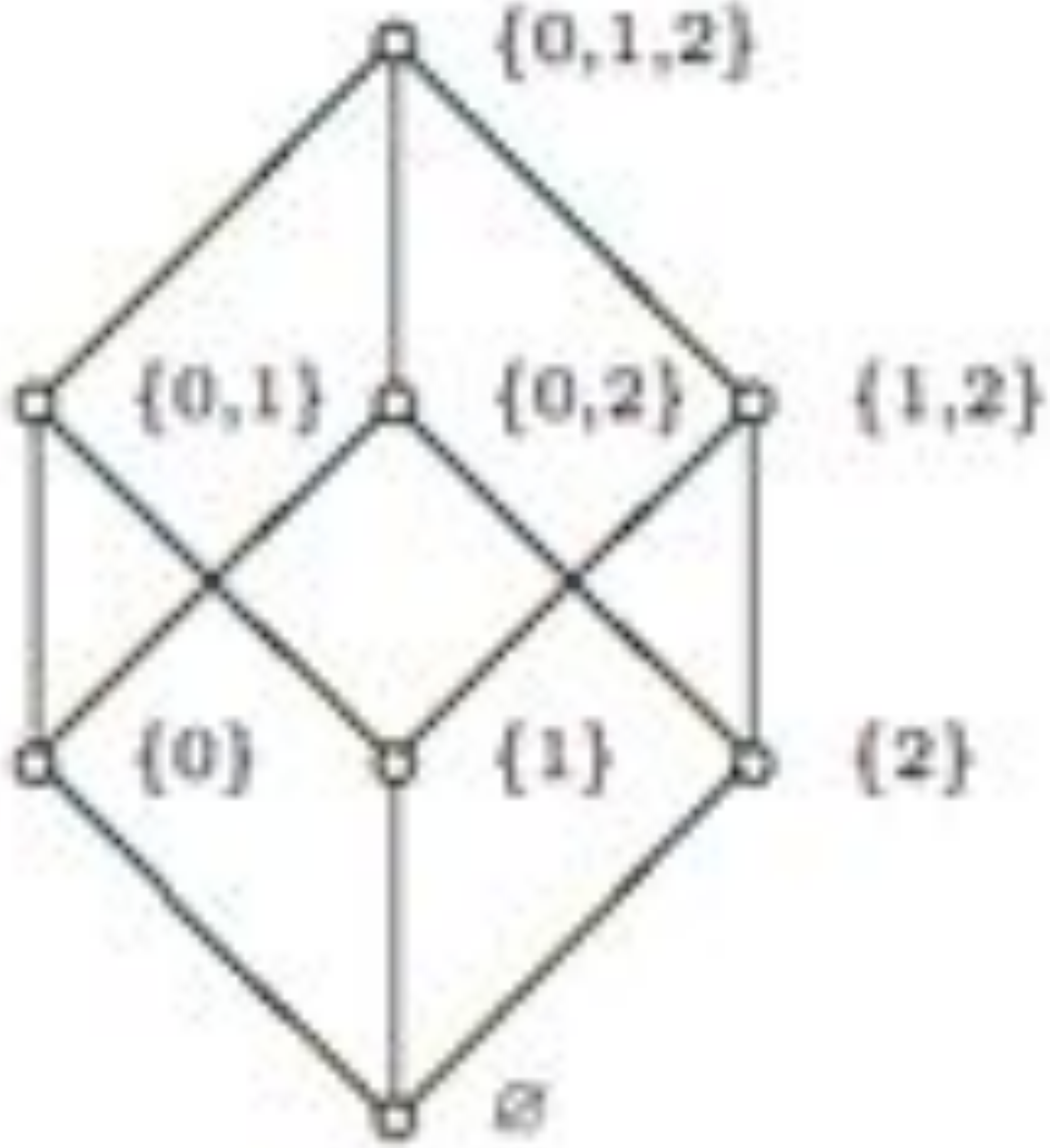
Context: Spatial Multi-agent Systems

- Agent spaces have
  - information
  - processes (apps)
  - agent spaces
- Agents can
  - run apps
  - **move apps and data**
  - **share opinions, facts, lies.**



Background

A **complete lattice** is a partially ordered set in which *all* subsets have both a supremum (join) and an infimum (meet).



Background

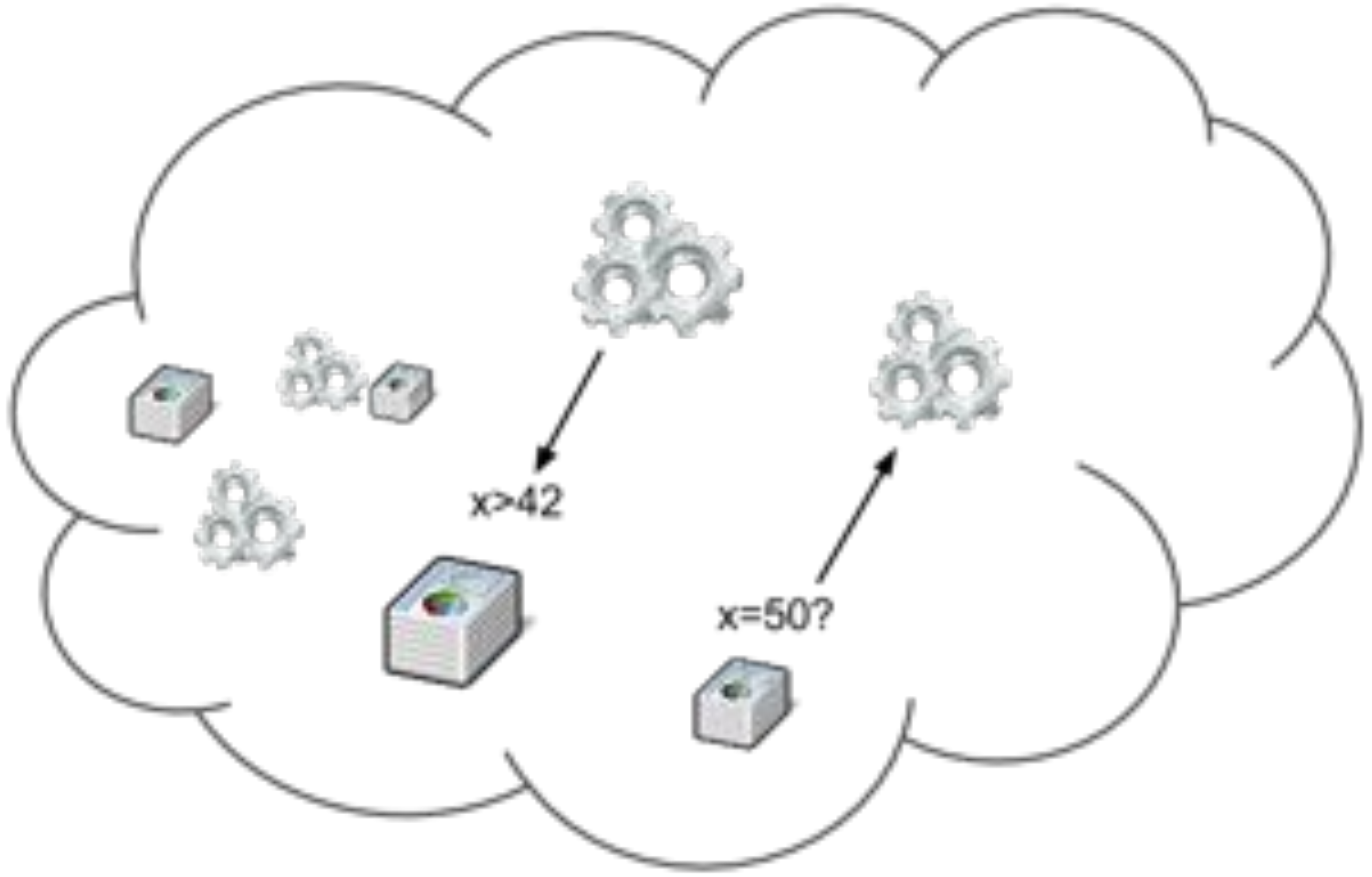
- **Epistemic logic** is a modal logic for reasoning about **knowledge** and **belief.** For $\Box \in \{\mathbf{B}, \mathbf{K}\}$
  - $\phi := \mathbf{p} \mid \phi \wedge \phi \mid \neg \phi \mid \Box_i \phi$
- $\mathbf{B_i}\ \phi$ means agent i believes ϕ.
- $\mathbf{K_i}\ \phi$ means agent i knows ϕ.
- Some axioms:
  - $\neg \mathbf{B_i}\ \mathbf{false}$, $(\mathbf{B_i}\phi \wedge \mathbf{B_i}(\phi \Rightarrow \psi)) \Rightarrow \mathbf{B_i}\psi$
  - $\mathbf{K_i}\phi \Rightarrow \phi$, $\mathbf{K_i}\phi \Rightarrow \mathbf{K_i}\mathbf{K_i}\phi$

Concurrent Constraint Programming (CCP)

CCP [Saraswat, Panagaden, Rinard]

- **(+)** Logic-based **process calculus** for **posting** information in a shared space (**store**).

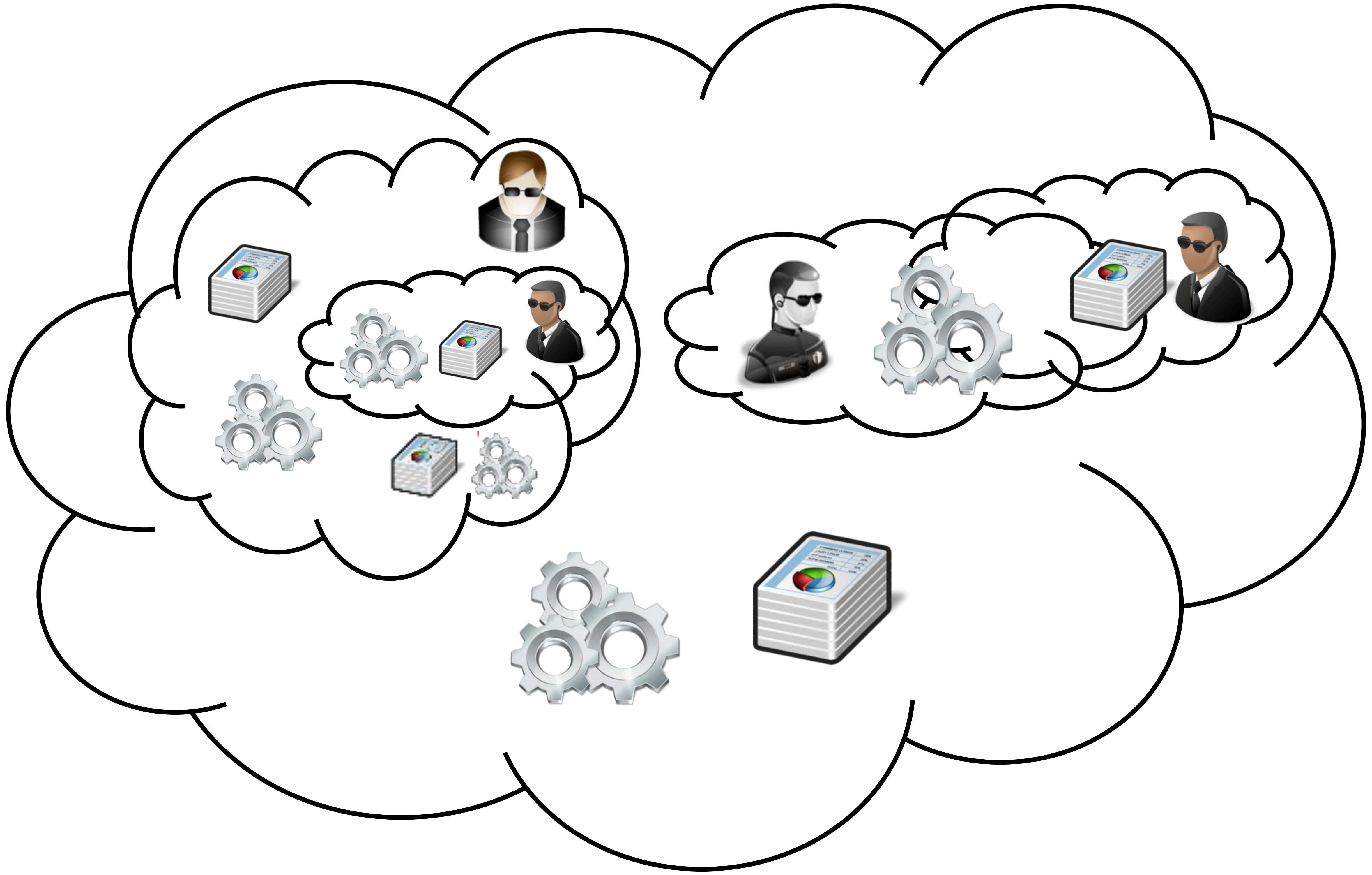
- **(-) But no agents, no local spaces.**



Spatial CCP

In **SCCP** [Knight, Panangaden et al]

- (+) Orthogonal extension CCP
- (+) Agents .
- (+) Local and nested spaces
- (+) Epistemic Information
- **(-) Space Mobility**



Processes, constraints and formulae

Calculus (Processes)	Lattice of Information = Constraint System (elements)	Logic (formulae)
post **tell(c)**	finite element **c**	primitive proposition **p**
parallel composition **P ‖ Q**	lub $\mathbf{c} \sqcup \mathbf{d}$	conjunction $\phi \wedge \Psi$
null process **0**	bottom $\bot$	**true**
abort	top $\top$	**false**
restriction $(\nu x)P$	cylindrification function $\exists_x(c)$	existential quantifier $\exists x \phi$
⋮	⋮	⋮
spatial construct $[P]_i$	space function $[\,c\,]_i$	belief modality $B(\phi)_i$
Mobility construct $\uparrow_i P$ $[R \parallel \uparrow_i P]_i \equiv [R]_i \parallel P$	**Extrusion function ?**	**Utterance modality?**

This Work

- **Contribution:** Algebraic structure to express extrusion and epistemic behaviors in spatially distributed multi-agent systems.
- **Outline:**
  - Background: Constraint System and Spatial Constraint Systems.
  - **Spatial Constraint Systems with Extrusion**.
  - The extrusion problem.
  - Properties of extrusion.
  - Application: A Logic for Utterances and Lies.

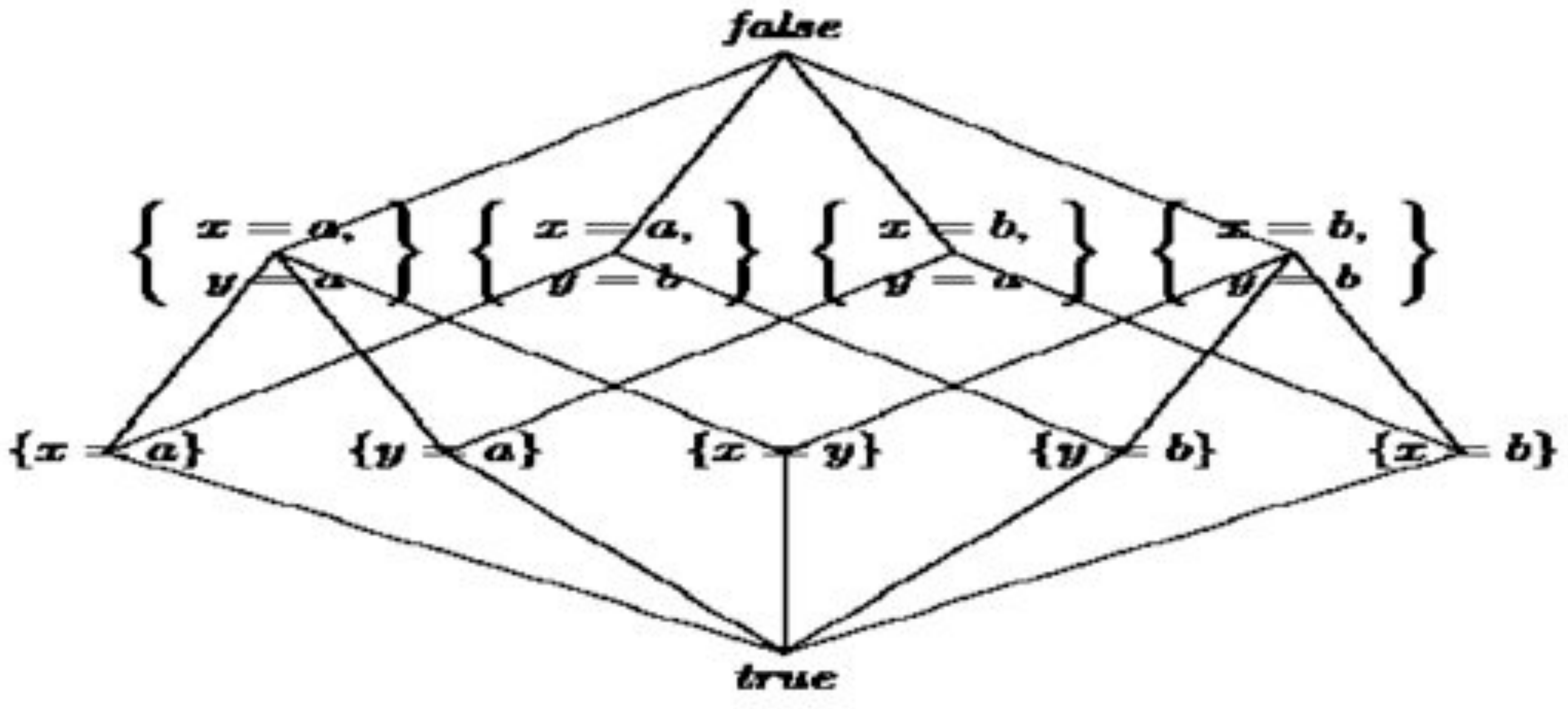
Constraint Systems

Def. A **constraint system (cs)** is a complete lattice C=**(Con,⊑)**. The elements of C are called **constraints**.

Think of

- **constraints** as propositions (partial information);
- the **join** ⊔ as conjunction (or composition).
- the **bottom** (true) is the empty information
- The **top** (false) is the join of all, possibly inconsistent, information.

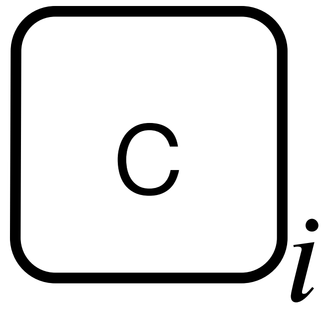
Example: Herbrand CS

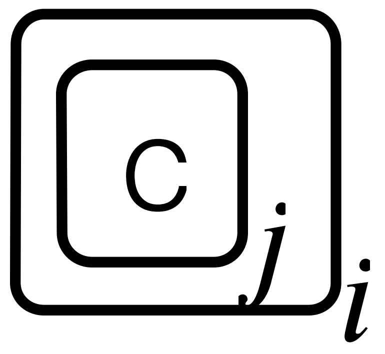


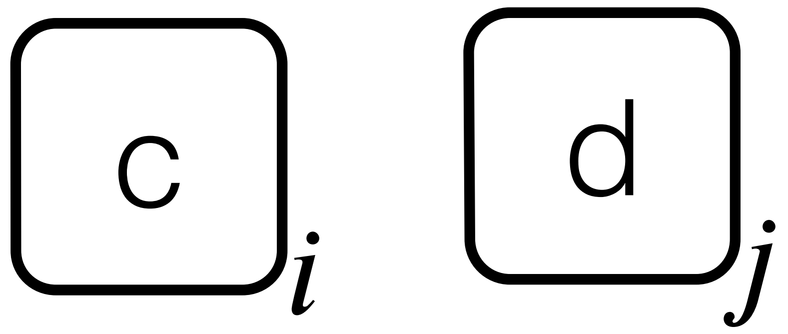
Herbrand Constraint System. Equality on terms with variables *x,y* and constants *a,b*.

Spatial CS

A **space function** $[.]_i$ is a self-map on the elements of $(Con, \sqsubseteq)$.

- $[c]_i$ means locality 

  - Epistemically: **agent i believes c.**
- $[[c]_j]_i$ means nesting 

- $[c]_i \sqcup [d]_j$ means composition 

Axioms for space functions

Definition. A **spatial constraint system (scs)** is a cs **(Con , ⊑)** equipped with self-maps $[.]_1 \ldots [.]_n$ that preserve finite joins (S.1 and S.2)

- S.1 $[\text{true}]_i = \text{true}$ (emptiness)
- S.2 $[c]_i \sqcup [d]_i = [c \sqcup d]_i$ (distribution)
- S.3 $c \sqsubseteq d$ implies $[c]_i \sqsubseteq [d]_i$ (monotonicity)

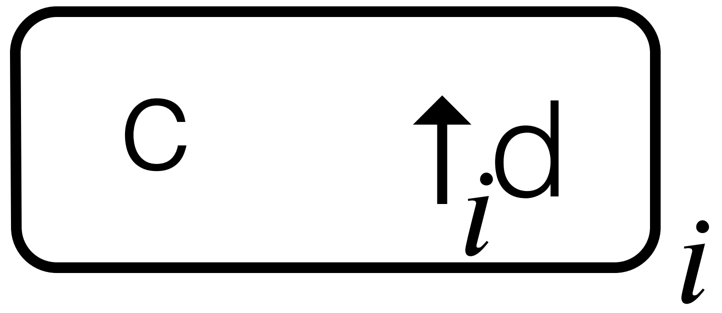
Some epistemic aspects

We allow the following to occur:

- **Inconsistency confinement**:
    - $[\textbf{false}]_i \neq \textbf{false}$
- **Freedom of opinion**: We may have
    - $[c]_i \sqcup [d]_j \neq \textbf{false}$ even if $c \sqcup d = \textbf{false}$
- **Information blindness** : We may have
    - $[\,\textbf{red}\,]_i = [\,\textbf{green}\,]_i$ even though $\textbf{red} \neq \textbf{green}$.

Spatial CS with Extrusion

- **Extrusion**: Moving information from within a space to the outside.



- **Utterance:** Statement made by an agent for others (possibly inconsistent, with the agent's own beliefs: e.g **a lie**)
- Utterance is the **epistemic interpretation** of extrusion.

Spatial CS with Extrusion: Definition

- **Extrusion** is the right inverse of space.
- Thus, an extrusion function $\uparrow_i$ is a map on $(Con , \sqsubseteq)$ such that

$$(E.1) \quad [\, \uparrow_i \mathbf{c} \,]_i = \mathbf{c}$$

- From E.1 and spatial axiom S.2: $[\, \mathbf{e} \sqcup \uparrow_i \mathbf{c} \,]_i = [\, \mathbf{e} \,]_i \sqcup \mathbf{c}$.

Def. A **spatial cs with extrusion (scs-e)** is a scs $(Con, \sqsubseteq, [.]_1, \ldots, [.]_n)$ equipped with maps $\uparrow_1(.), \ldots, \uparrow_n(.)$ that satisfy E.1.

Derived Constructs

- $c \rightarrow d$ is $\sqcap\{ e \mid e \sqcup c \sqsupseteq d \}$ (**Heyting implication**).
- $\sim c$ is $c \rightarrow \text{false}$ (**pseudo complement**).
- Think of $c \rightarrow d$ as a little **process** that on input c produces d.
- In fact, $c \sqcup (c \rightarrow d) \sqsupseteq d$ (MP, **Modus Ponens**).

Lies on lattices

A **lie** *is not necessarily a false statement but rather a statement that it is inconsistent with what its author believes to be true.*

- A lie ~c by agent i: $[\, c \sqcup \uparrow_i {\sim}c \,]_i$
- In fact $[\, c \sqcup \uparrow_i {\sim}c \,]_i \sqsupseteq [\, c \,]_i \sqcup {\sim}c$

$\boxed{c \quad \uparrow_i {\sim}c}_{\,i}$

Communication Examples.

- An intrusive process:
  - Let **p** $= c \rightarrow \uparrow_j [c]_i$ (think of **p** as conditional program)
  - Consider $[\, e \sqcup \uparrow_i [\, \mathbf{p} \,]_j \,]_i \sqcup [\, c \,]_j$.
  - We derive $[\, e \,]_i \sqcup [\, c \sqcup \mathbf{p} \,]_j$ (using E.1 and S.2)
  - We can derive $[\, e \sqcup c \,]_i$ (using MP, S.2 and E.1)

Roadmap

- We have defined spatial constraint systems with **extrusion.**
- **Next:**
  - **The Extrusion Problem**
  - Properties of Space and Extrusion.
  - A Logic with Utterance.

Extrusion Problem

Problem: Given a spatial constraint system **(Con,⊑,[.]1,...,[.]n)** construct maps $\uparrow_1(.),\ldots,\uparrow_n(.)$ that satisfy E.1: $[\ \uparrow_i c\ ]_i = c$.

Th. Suppose that $[.]_i$ is a ***surjective and continuous function*** then $\uparrow_i : c \longmapsto \bigsqcup\{ d \mid [d]_i = c\}$ satisfies E.1.

Distributive Extrusion

Extrusion distribute **locally/subjectively**: Let $c \approx_i d$ iff $\mathbf{[c]}_i \approx_i \mathbf{[d]}_i$

Th. $\uparrow_i(\text{true}) \approx_i \text{true}$ and $\uparrow_i(c \sqcup d) \approx_i \uparrow_i(c) \sqcup \uparrow_i(d)$

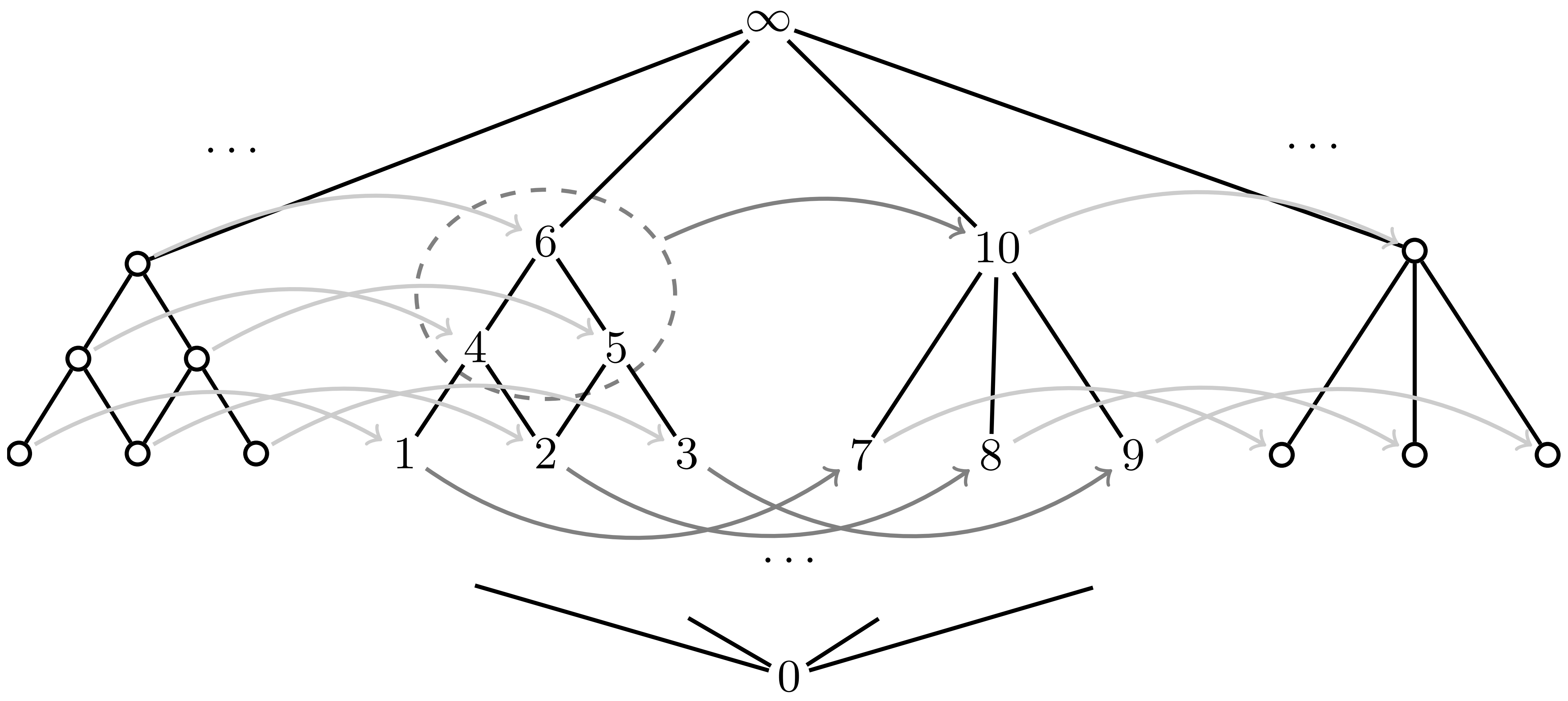
Extrusion **may not** distribute **globally/objectively**; may fail to satisfy

E.2: $\uparrow_i(\text{true}) = \text{true}$ and ***E.3:*** $\uparrow_i(c \sqcup d) = \uparrow_i(c) \sqcup \uparrow_i(d)$

Distributed Extrusion Problem

Problem: Given a spatial constraint system **$(Con, \sqsubseteq, [.]_1, \ldots, [.]_n)$**, find $\uparrow_1(.), \ldots, \uparrow_n(.)$ that satisfy E.2 and E.3.

Not always possible:



Th. If **$[.]_i$** is ***surjective and meet complete*** then the map $\uparrow_i : c \longmapsto \bigsqcap\{ d \mid [d]_i = c\}$ satisfies E.2 and E.3.

Consequences of Extrusion

- Extrusion **prevents** inconsistency confinement.

  Th. $[\,\textbf{false}\,]_i = \textbf{false}$

- (But **freedom of opinion** and **information blindness** can still occur. We may have:

  - $[c]_i \sqcup [d]_j \neq \textbf{false}$ even if $c \sqcup d = \textbf{false}$,
  - $[c]_i = [d]_j$ even if $c \neq d$)

Embeddings & Galois Connection

If $\uparrow_i$ is distributed then $c \sqsubseteq d \;\Leftrightarrow\; \uparrow_i(c) \sqsubseteq \uparrow_i(d)$

If $[\![.]\!]_i$ is injective then $c \sqsubseteq d \Leftrightarrow [\![c]\!]_i \sqsubseteq [\![d]\!]_i$

If $\uparrow_i(c) = \sqcap\{\, d \mid [\![d]\!]_i = c\}$ then $\uparrow_i(c) \sqsubseteq d \Leftrightarrow c \sqsubseteq [\![d]\!]_i$

Utterance the right inverse of Belief

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid B_i\varphi \mid U_i\varphi$$

$$\mathcal{O}_i(\varphi) \stackrel{\text{def}}{=} B_i(\varphi \wedge U_i(\varphi)) \qquad \mathcal{O}_i(\varphi) \Leftrightarrow (B_i\varphi) \wedge \varphi$$

$$\mathcal{H}_i(\varphi) \stackrel{\text{def}}{=} B_i(\neg\varphi \wedge U_i(\varphi)) \qquad \mathcal{H}_i(\varphi) \Leftrightarrow (B_i\neg\varphi) \wedge \varphi$$

$\varphi \Rightarrow B_i\psi$ if and only if $U_i\varphi \Rightarrow \psi$

Concluding Remarks

- We presented a **lattice structure** to express extrusion and epistemic behaviors in spatially distributed multi-agent systems.
- The structure can be used to derive a **belief logic with utterance**.
- Extrusion/Utterance viewed as **the right inverse** of Space/Belief.

Thank you